

The gravitino problem in supersymmetric warm inflation

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The warm inflation paradigm considers the continuous production of radiation during inflation due to dissipative effects. In its strong dissipation limit, warm inflation gives way to a radiation dominated Universe. High scale inflation then yields a high reheating temperature, which then poses a severe gravitino overproduction problem for the supersymmetric realisations of warm inflation. In this paper we show that in certain class of supersymmetric models the dissipative dynamics of the inflaton is such that the field can avoid its complete decay after inflation. In some cases, the residual energy density stored in the field oscillations may come to dominate over the radiation bath at a later epoch. If the inflaton field finally decays much later than the onset of the matter dominated phase, the entropy produced in its decay may be sufficient to counteract the excess of gravitinos produced during the last stages of warm inflation.

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I. INTRODUCTION

The flatness required of inflationary potentials, to satisfy density perturbation constraints, most commonly relies of Supersymmetry to protect against radiative corrections. Although SUSY is hugely successful in building models of inflation, associated with it are also some problems. One of these is the gravitino overproduction problem. Very simply the problem is, ending inflation at too high a temperature can lead to a gravitino abundance that is prohibited by nucleosynthesis constraints when we consider a massive unstable gravitino (in this paper we will mainly discuss the case of the massive unstable gravitino with the mass of $m_{3/2} \sim \mathcal{O}(1)$ TeV). One alternative is to end inflation at a lower temperature, but there are also advantages to a high temperature exit from inflation, notably it can lead to effective leptogenesis [1, 2].

There are two distinct dynamical realizations of inflation, cold and warm inflation. Cold inflation is the standard scenario in which the inflaton is assumed to be non-interacting during inflation, and thus there is no particle production during inflation, so the Universe supercools [3]. Only after inflation, interactions of the inflaton with other fields is assumed significant, and a reheating phase occurs when the vacuum energy used to drive inflation is

converted into particles that forms the subsequent radiation dominated phase. In the alternative warm inflation picture, the inflaton interacts with other fields during the inflation phase, which leads to particle production concurrent with inflationary expansion [4] (for recent reviews please see [5–7]). The presence of a radiation energy density is not inconsistent with the General Relativity requirements for realizing inflation, which only requires that the vacuum energy dominates the energy density in the Universe. Thus in the warm inflation picture, the presence of radiation during inflation implies this phase smoothly ends into a radiation dominated phase without a distinctively separate reheating phase. This offers an alternative dynamic solution to the graceful exit problem of inflation. The presence of radiation then implies the fluctuation of the inflaton, which are the primordial seeds of density perturbations, are now thermal [8–10], rather than the quantum fluctuations that occur in cold inflation.

The equations of motion for the inflaton field and for the radiation density in the presence of a dissipation mechanism are

$$\ddot{\phi} + 3H(1 + Q)\dot{\phi} + V' = 0, \quad (1)$$

and

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2, \quad (2)$$

where $Q \equiv \Upsilon/3H$, Υ is the dissipative coefficient, over-dots stand for time derivative, and $' \equiv \frac{d}{d\phi}$. Throughout the paper we use natural units $c = \hbar = k_B = 1$ and Newton's gravitational constant is $8\pi G = m_P^{-2}$, where $m_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass.

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Since interactions are important during warm inflation, if the fields interacting with the inflaton are at high temperature, then it is difficult to control the thermal loop corrections to the effective potential that is needed to maintain the very flat potential required for inflation [11, 12]. However, if the fields interacting with the inflaton are at low temperature, then supersymmetry can be used to cancel the quantum radiative corrections, and maintain a very flat inflaton potential. In this paper we consider a class of supersymmetric models for which the dissipation coefficient Υ has been computed in the equilibrium approach [11] for the low-temperature regime [13, 14]. The dissipation mechanism is based on a two-stage process [15]. The inflaton field couples to heavy bosonic fields, χ and fermionic fields ψ_χ , which then decay to light degrees of freedom. These light degrees of freedom thermalize to become radiation. The simplest superpotential containing such an interaction structure is

$$W = g\Phi X^2 + hXY^2, \quad (3)$$

where Φ , X and Y denote superfields, and ϕ , χ and y refer to their bosonic components. Such an interaction structure is common in many particle physics SUSY models during inflation, the field y and its fermionic partner \bar{y} remain massless or very light, whereas the field χ and its fermion partner ψ_χ obtain their masses through their couplings to ϕ , namely $m_{\psi_\chi} = m_\chi = g\phi$. The regime of interest is when $m_\chi, m_{\psi_\chi} > T > H$, and this defines what is usually referred to as the low-temperature regime. For this regime the dissipation coefficient, when the superfields X and Y are singlets, is found to be [13, 14]

$$\Upsilon \simeq 0.64 g^2 h^4 \left(\frac{g\phi}{m_\chi} \right)^4 \frac{T^3}{m_\chi^2}, \quad (4)$$

where T is the temperature of the radiation bath, $\rho_r = C_r T^4$, where $C_r = \pi^2 g_*/30$ and g_* is the number of relativistic degrees of freedom. The above dissipative coefficient is calculated under the adiabatic approximation, and associated with it are consistency conditions which ensure that the microscopic dynamics is faster than the macroscopic motion [11],

$$\Gamma_{\chi (\psi_\chi)} \gg \frac{\dot{\phi}}{\phi}, H. \quad (5)$$

In supersymmetric theories $C_r \simeq 70$. However, the superfields X and Y may belong to large representations of a GUT group. In that case, the dissipation coefficient picks up an extra factor $\mathcal{N} = \mathcal{N}_X \mathcal{N}_{\text{decay}}^2$, where \mathcal{N}_X is the multiplicity of the X superfield and $\mathcal{N}_{\text{decay}}$ is the number of decay channels available in X 's decay. Typically, having enough dissipation during inflation requires large multiplicities, $\mathcal{N}_X \sim \mathcal{N}_{\text{decay}} \sim O(100)$. In that respect, string theory can be a natural place for warm inflation due to the presence of large numbers of moduli fields [16].

Following this approach, it has been recently shown [17, 18] that chaotic and hybrid inflation models may

support some 50 to 60 e -foldings of warm inflation in the strong dissipative regime, but such models can lead to an overproduction of gravitinos [19]. In the context of hilltop models, it was found in Ref. [20] that although warm inflation agrees with current observations the temperature of the radiation at the end of inflation exceeds the current bounds on thermal gravitino production for massive gravitino with its mass $m_{3/2} = \mathcal{O}(\text{TeV})$.

In order to dilute the excess of gravitinos thermally produced towards the end of warm inflation, it is possible to argue that the necessary entropy production owes to the decay of the inflaton field, which comes to dominate the Universe at a later epoch. However, if the ratio $Q \gg 1$, the inflaton field decays completely right after inflation and no later entropy production can be attributed to it. It is worth emphasizing though, that Υ , as given in Eq. (4), is time-dependent. Moreover, it *always* falls faster than the Hubble parameter during the radiation dominated epoch that follows after inflation: $\Upsilon \propto a^{-3}$ whereas $H \propto a^{-2}$. Therefore, if Q is not too large, it is plausible that soon after inflation the system moves into the weak dissipation and hence the inflaton field does not decay completely until a later epoch. Once in the weak dissipation regime, the average scalar density of the field decreases as a^{-3} , hence it may come to dominate over the radiation density thus driving a late matter-dominated epoch. This epoch is terminated by the perturbative decay of the inflaton field. The entropy produced by this decay may be sufficient to dilute the excess of gravitinos. After this, the inflaton decays completely before the Big-bang nucleosynthesis epoch and the thermal bath of the hot big bang is recovered¹.

II. FIELD EVOLUTION

In order to investigate the field evolution during inflation and its subsequent oscillatory phase we consider a class of hilltop models with the scalar potential given by

$$V = V_0 f(\phi), \quad (6)$$

where V_0 is a characteristic density scale and f is a dimensionless function with a maximum at $\phi = 0$ and a minimum at $\phi = \phi_v$, where it vanishes. We parametrise the field's expectation value by the dimensionless quantity δ , defined by

$$\phi \equiv \phi_v(1 - \delta). \quad (7)$$

In the following we use subscripts “*”, “ e ” and “ o ” to denote the time when cosmological scales exit the horizon,

¹ When we do not consider the nonthermal gravitino production in supergravity, the most conservative lower bounds on the reheating temperature, $T_R > 3 - 4$ MeV, comes from the insufficient thermalisation of the neutrino background, which changes the ⁴He abundance [21].

the end of inflation, and the onset of the field oscillations respectively. We consider the two-stage dissipation mechanism described in the introduction and take

$$\Upsilon \simeq C_\phi T^3 \dot{\phi}^{-2}. \quad (8)$$

where² $C_\phi = 0.64 h^4 \mathcal{N}$.

A. Inflation and first reheating

We are interested in the amount of expansion that follows after the observable Universe exits the horizon, and so we require that warm inflation is supported for around 50 e -foldings. In the warm inflation paradigm, the slow-roll equations that apply during inflation are

$$\dot{\phi} \simeq -\frac{V'}{3H(1+Q)} \quad \text{and} \quad \rho_r \simeq \frac{\Upsilon \dot{\phi}^2}{4H}. \quad (9)$$

These equations hold for as long as the so-called *modified* slow-roll parameters, given by [23]

$$\epsilon = \frac{\epsilon_\phi}{(1+Q)}, \quad (10)$$

$$\eta = \frac{\eta_\phi}{(1+Q)}, \quad (11)$$

and

$$\epsilon_{HY} = \frac{1}{(1+Q)} \frac{V'}{3H^2} \frac{\Upsilon'}{\Upsilon}, \quad (12)$$

are sufficiently small. In the above, ϵ_ϕ and η_ϕ are the slow-roll parameters in the absence of a dissipative mechanism

$$\epsilon_\phi = \frac{m_P^2}{2} \left(\frac{V'}{V} \right)^2, \quad \eta_\phi = m_P \frac{V''}{V}. \quad (13)$$

Because the radiation density ρ_r remains subdominant until the end of inflation, we may obtain the evolution of H during inflation simply by integrating $d(H^2) \simeq \frac{dV}{3m_P^2}$. Using Eq. (6), the Hubble parameter when cosmological scales exit the horizon is

$$H_* \simeq H_0 f_*^{1/2}, \quad (14)$$

where H_0 is defined by $V_0 \equiv 3H_0^2 m_P^2$. In the strong dissipative regime of warm inflation, the accelerated expansion lasts until the radiation density, constantly produced owing to the dissipation mechanism, catches up with $V(\phi)$. Somewhat before this time, the Hubble parameter slightly deviates from $H \simeq H_0 f^{1/2}$. If we neglect

the kinetic density of the field, which in the strong dissipation regime remains subdominant during inflation, the Hubble parameter when the radiation density catches up with $V(\phi)$ is well approximated by

$$H_e \simeq \sqrt{2} H_0 f_e^{1/2}. \quad (15)$$

Although the accelerated expansion finishes at this time, quasi-de Sitter inflation ends somewhat earlier. Nevertheless, the amount of expansion that follows from that moment until the end of accelerated expansion is negligible. Therefore, we approximate the number of e -foldings N_* by considering that the exponential expansion finishes at the time of potential-radiation equality. Also at this time, the source term $\Upsilon \dot{\phi}^2$ in Eq. (2) starts becoming subdominant, and the radiation density separates from its attractor solution and starts scaling as $\rho_r \propto a^{-4}$. On its part, and provided that $Q_e > 1$, the scalar field still obeys its slow-roll equation and continues transforming its potential energy into radiation. Consequently, immediately after the time of equality the scalar density becomes subdominant and the Universe becomes radiation dominated. We define the *first reheating temperature* T_{R_1} as the temperature at the time of equality, namely

$$T_{R_1} \equiv T_e \simeq C_r^{-1/4} (V_0 f_e)^{1/4}. \quad (16)$$

B. Onset of inflaton oscillations

Because Υ depends on the temperature of the radiation, in the radiation dominated phase Υ evolves in a manner different than during inflation. As a result, the inflaton field loses energy at a different rate. Here we compute the potential density of the field when the field ceases to follow its attractor equation. This time, owing to the dissipation term, is determined by the condition

$$\Upsilon_o \sim |V_o''|^{1/2}. \quad (17)$$

Integrating the slow-roll equation $\dot{\phi} \simeq |V'|/\Upsilon$ from the end of inflation until the onset of the fast-roll motion we obtain the integral equation that determines ϕ_o

$$I_\phi = \int_{\phi_e}^{\phi_o} \frac{d\phi}{V' \dot{\phi}^2} \simeq - \int_{t_e}^{t_o} \frac{dt}{C_\phi T^3} = I_t, \quad (18)$$

where we used $\Upsilon = C_\phi T^3 / \dot{\phi}^2$. Using that in this stage the Universe is radiation dominated, i.e. $H \simeq H_e (a_e/a)^2$ and $T \simeq T_{R_1} (a_e/a)$, the integral I_t may be approximated by

$$I_t = - \int_{a_e}^{a_o} \frac{da}{(aH) C_\phi T^3} \simeq \frac{1}{5 C_\phi H_e T_{R_1}^3} \left[1 - \left(\frac{a_o}{a_e} \right)^5 \right]. \quad (19)$$

The ratio a_o/a_e can be estimated as follows. Using the definition of Υ and that $T \propto a^{-1}$ it is straightforward to obtain the relation

$$\Upsilon_o \simeq \Upsilon_e \left(\frac{a_e}{a_o} \right)^3 \frac{(1 - \delta_e)^2}{(1 - \delta_o)^2}, \quad (20)$$

² This should be small in case of MSSM [22].

where Υ_e can be obtained from the attractor equations and Υ_o is determined by Eq. (17). The redshift factor (a_o/a_e) is then approximated by

$$\left(\frac{a_o}{a_e}\right)^5 \sim \frac{C_\phi^{20/21} (f'_e)^{10/7} (1 - \delta_e)^{10/7} H_0^{10/21} m_P^{25/21}}{2C_r^{5/7} |f''_o|^{5/6} f_e^{5/14} (1 - \delta_o)^{10/3} \phi_v^{5/3}}, \quad (21)$$

where $' \equiv \frac{d}{d\delta}$. Substituting this expression into Eq. (19) we can solve Eq. (18) numerically and find δ_o .

C. Second reheating temperature

In order to simplify our analytical treatment we assume in this section that at $a = a_o$ the field starts performing fast oscillations about its vev. In such case its average density $\langle \rho_\phi \rangle$ can be readily computed [24]. Although this assumption naturally results in an error estimating the scalar density of the field, in the Appendix we explain how this can be corrected, thus obtaining an accurate estimate of the average scalar density at late times. Assuming then a sudden transition to the stage of fast oscillations, the average density of the field is determined by

$$\langle \dot{\rho}_\phi \rangle \simeq -3H(1 + Q)\langle \rho_\phi \rangle. \quad (22)$$

Although at the beginning of the oscillations it may be $Q > 1$, we keep the term $3H$ in the above as this becomes the dominant one when the system moves into the weak dissipation regime. Using that $H \simeq H_o \left(\frac{a_o}{a}\right)^2$ and that $\langle \phi \rangle = \phi_v$, and hence $\Upsilon \simeq \Upsilon_o \left(\frac{a_o}{a}\right)^3$, the average density of the field can be readily obtained

$$\langle \rho_\phi \rangle \simeq (\rho_\phi)_o \left(\frac{a_o}{a}\right)^3 \exp \left\{ -3Q_o \left(1 - \frac{a_o}{a}\right) \right\}, \quad (23)$$

where Q_o can be expressed in terms of δ_e and δ_* after using Eq. (20). From the above expression it follows that $\langle \rho_\phi \rangle \propto a^{-3}$ a few Hubble times after the onset of oscillations. Thus, provided that Q_o is not large, the inflaton may come to dominate the Universe at a later time well before nucleosynthesis. We denote by t_{eq} the time when the average scalar density $\langle \rho_\phi \rangle$ catches up with ρ_r . The subsequent matter-dominated phase must be sufficiently long so that the entropy produced when the inflaton field decays completely at $t = t_{\text{dec}}$ is enough to dilute the overproduction of gravitinos. With the complete decay of the inflaton at $t = t_{\text{dec}}$, which we consider a free parameter, the Hot Big Bang evolution is recovered. The second reheating temperature T_{R_2} is then

$$T_{R_2} \sim C_r^{-1/4} \langle \rho_\phi \rangle_{\text{eq}}^{1/4} \left(\frac{a_{\text{eq}}}{a_{\text{dec}}} \right)^{3/4}, \quad (24)$$

which depends on δ_e and on the model parameters. Obviously, to estimate this temperature we first need to have an estimate for the quantities δ_* and δ_e .

III. OBSERVATIONAL CONSTRAINTS

The purpose of this section is to determine δ_* and δ_e , employed to compute T_{R_2} , in terms of the model parameters. The *raison d'être* of this is to link the reheating temperatures T_{R_2} , computed according to Eq. (24), to observable quantities like the spectral index or its running, in turn determined by the model parameters. By doing this we manage to identify the interval of temperatures T_{R_2} corresponding to certain range of values of the spectral index (see Fig. 2).

The predicted amplitude of the curvature perturbation spectrum in the strong dissipative regime of warm inflation is given by

$$\mathcal{P}_{\mathcal{R}} \simeq \widetilde{\mathcal{P}}_{\mathcal{R}} \mathcal{P}_{\mathcal{R}}^{(c)} \quad (25)$$

where

$$\widetilde{\mathcal{P}}_{\mathcal{R}} \simeq \left(\frac{3H^3}{2\pi V'} \right)^2 (1 + Q)^{5/2} \frac{T}{H} \quad (26)$$

has been known for sometime [10], whereas the correction $\mathcal{P}_{\mathcal{R}}^{(c)}$ was computed only recently [25]. The correction term has a range of behavior, depending on the details of the dynamics of the radiation energy density component. We have examined this range, but will not explore these details here. The purpose of this paper is to highlight a general mechanism for treating gravitino production within warm inflation, and for that we will focus on one particular form of the correction [25, 26] for the dissipation coefficient in Eq. (4),

$$\mathcal{P}_{\mathcal{R}}^{(c)} \simeq \frac{1 + a_0 Q^2 + a_1 Q^3}{\sqrt{1 + Q}}, \quad (27)$$

where $a_0 \simeq 0.662$ and $a_1 \simeq 3.26 \times 10^{-4}$.

Using Eqs. (6), (9) and (14), and writing Q in terms of C_ϕ , H_0 and ϕ the above may be recast as

$$\mathcal{P}_{\mathcal{R}} = \mathcal{P}_{\mathcal{R}}(C_\phi, H_0, \phi_*, \beta_i), \quad (28)$$

which can be solved numerically to determine ϕ_* . Here β_i denotes the rest of the model parameters which the amplitude of the spectrum may depend on.

In the strong dissipative regime warm inflation gives way to a radiation dominated Universe. Neglecting the kinetic density of the field, the end of the accelerated expansion occurs when the radiation density catches up with the scalar potential, i.e. when

$$V_e = (\rho_r)_e. \quad (29)$$

As mentioned earlier, the amount of quasi-de Sitter inflation is well approximated by considering that this finishes at $\phi = \phi_e$. To determine ϕ_e we rewrite the above condition expressing V_e in terms of δ_e and using the attractor equation for (ρ_r) . The field value ϕ_e is then determined by

$$\frac{(f'_e)^{8/9} (1 - \delta_e)^{8/9}}{f_e} \simeq \frac{2^{11/9} C_\phi^{4/9}}{3^{1/9} C_r^{1/3}} \left(\frac{H_0}{m_P} \right)^{2/9}. \quad (30)$$

By simple inspection, it is clear that solving for ϕ_* and ϕ_e from Eqs. (28) and (30) is, in general, not amenable to analytical treatment. Owing to this, we proceed alternatively by solving H_0 and C_ϕ in terms of δ_* and δ_e , i.e.

$$H_0 = H_0(\delta_*, \delta_e) \quad \text{and} \quad C_\phi = C_\phi(\delta_*, \delta_e). \quad (31)$$

The amount of exponential expansion that follows from horizon crossing is given by $N \simeq \int \frac{H}{|V'|} d\phi$. Although this approximation is valid in the strong dissipative regime (i.e. $Q \gg 1$), by solving numerically the inflationary stage we find that this formula still constitutes a good approximation to the number of e -foldings for $Q_* \gtrsim 2$. Writing $H \simeq H_0 f^{1/2}$ (which holds until the end of inflation) and using Eq. (30), the amount of expansion after the observable Universe exits the horizon is given by

$$N_* \simeq \frac{(f'_e)^{8/7} (1 - \delta_e)^{8/7}}{2^{16/7} f_e^{9/7}} \int_{\delta_e}^{\delta_*} \frac{f^{2/7} d\delta}{(1 - \delta)^{8/7} (f')^{1/7}}. \quad (32)$$

Keeping fixed the number of e -foldings N_* it is possible to solve for δ_* in terms of δ_e by integrating numerically the above equation. Our strategy now consists of employing this parametrisation $\delta_* = \delta_*(\delta_e)$ and Eq. (31) to express the observable quantities in terms of δ_e .

In the absence of the corrected part of the perturbation spectrum, i.e. $\mathcal{P}_{\mathcal{R}}^{(c)} = 1$, the spectral index of the perturbation spectrum, defined as $n_s - 1 = \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$, is given by [23]

$$\widetilde{n}_s - 1 \simeq \frac{1}{1 + Q_*} \left(-(2 - 5A)\epsilon_\phi - 3A\eta_\phi + (2 + 4A)\sigma_\phi \right), \quad (33)$$

where

$$\sigma_\phi = m_P^2 \frac{V'}{V\phi} \quad \text{and} \quad A = \frac{Q}{1 + 7Q}. \quad (34)$$

Using the slow-roll equations, the ratio Q at the time of horizon crossing is given by may be rewritten as follows

$$Q_* \simeq \frac{C_r^{-3/7} (f'_*)^{6/7} C_\phi^{4/7}}{2^{6/7} 3^{1/7} (1 - \delta_*)^{8/7} f_*^{5/7}} \left(\frac{m_P}{\phi_v} \right)^2 \left(\frac{H_0}{m_P} \right)^{2/7}. \quad (35)$$

Upon including the correction $\mathcal{P}_{\mathcal{R}}^{(c)}$, the spectral index of the corrected spectrum is given by

$$n_s = \widetilde{n}_s + \frac{d \ln \mathcal{P}_{\mathcal{R}}^{(c)}}{d \ln k}, \quad (36)$$

where

$$\frac{d \ln \mathcal{P}_{\mathcal{R}}^{(c)}}{d \ln k} = \frac{d \ln \mathcal{P}_{\mathcal{R}}^{(c)}}{d Q} A (10\epsilon_\phi - 6\eta_\phi + 8\sigma_\phi). \quad (37)$$

Using the combined WMAP+BAO+SN data and for negligible tensor perturbations, current observations constrain the spectral index to $n = 0.963 \pm 0.014$ at the 1- σ level [27].

Also interesting for observational purposes is the running of the spectral index, defined as

$$n'_s = \widetilde{n}_s' + n_s'^{(c)}, \quad (38)$$

where \widetilde{n}_s' can be written in terms of slow-roll parameters [7] and

$$n_s'^{(c)} = \frac{d^2 \ln \mathcal{P}_{\mathcal{R}}^{(c)}}{d \ln k^2} = \frac{d^2 \ln \mathcal{P}_{\mathcal{R}}^{(c)}}{d Q^2} (10\epsilon_\phi - 6\eta_\phi + 8\sigma_\phi)^2 A^2 + \frac{d \ln \mathcal{P}_{\mathcal{R}}^{(c)}}{d Q} \left[\frac{(10\epsilon_\phi - 6\eta_\phi + 8\sigma_\phi)^2}{(1 + 7Q)^2} + 10\epsilon_\phi - 6\eta_\phi + 8\sigma_\phi \right] A. \quad (39)$$

In the absence of tensor perturbations and using the combined WMAP+BAO+SN data, the running is constrained to the interval $n'_s = -0.034 \pm 0.026$ at the 1- σ level [27].

IV. CONSTRAINTS ON GRAVITINO PRODUCTION

Here we discuss production processes of gravitino and observational constraints on its abundance. First of all, the gravitino is produced through the thermal scattering among the standard particles such as quarks and gluons. Then the yield value, $Y_{3/2} \equiv n_{3/2}/s$ of the thermally-produced gravitino is approximately given by [28–32]

$$Y_{3/2} \simeq 2 \times 10^{-14} \left(\frac{T_R}{10^8 \text{ GeV}} \right) \left(1 + \frac{m_{\tilde{g}}^2}{3m_{3/2}^2} \right) \quad (40)$$

with $m_{\tilde{g}}$ being the gluino mass. Note that this yield value is proportional to the reheating temperature. For massive unstable gravitinos with $m_{3/2} \gg 10^2 \text{ GeV} \sim \mathcal{O}(m_{\tilde{g}})$, the second term in the second bracket is negligible. The energetic products by the decay of gravitino are so dangerous that the light element abundances are modified by their annihilations and productions through scattering processes. When we consider the case of $m_{3/2} \sim \mathcal{O}(1) \text{ TeV}$, to agree with the observational light element abundances we get upper bounds on the gravitino abundance to be $Y_{3/2} \lesssim 10^{-16}$ and $Y_{3/2} \lesssim 10^{-14}$ for a branching ratio decaying into hadrons, $B_h = 1$ and $B_h = 10^{-3}$, respectively [29, 33]. The latter value of B_h is a reasonable lower limit on the hadronic branching ratio [29]. Then we obtain the upper bound on the reheating temperature in turn through Eq. (40) to be $T_R \lesssim \mathcal{O}(10^6) \text{ GeV}$ ($T \lesssim \mathcal{O}(10^8) \text{ GeV}$) with a hadronic branching ratio $B_h = 1$ ($B_h = 10^{-3}$) [29, 33] (see Fig. 44 and Fig. 45 of Ref. [29]).

On the other hand, for much larger masses, $m_{3/2} \gtrsim 70 \text{ TeV}$ with $B_h = 1$ (or $m_{3/2} \gtrsim 20 \text{ TeV}$ with $B_h = 10^{-3}$), we have another type of upper bound on the abundance of gravitino which produces the Lightest SUSY Particle

(LSP) dark matter as a decay product. This is estimated to be

$$Y_{3/2} = Y_{\text{LSP}} \lesssim 4 \times 10^{-12} \left(\frac{\Omega_{\text{LSP}} h^2}{0.12} \right) \left(\frac{m_{\text{LSP}}}{10^2 \text{ GeV}} \right)^{-1}. \quad (41)$$

By adopting the upper bound on the density parameter of LSP, $\Omega_{\text{LSP}} h^2 \lesssim 0.12$ [27] with h the reduced Hubble parameter, and a typical scale of the LSP mass ($m_{\text{LSP}} \sim 10^2$ GeV), this gives us the upper bound on the reheating temperature $T_R \lesssim 2 \times 10^{10}$ GeV. We see that this is much milder than that of BBN for the thermally-produced gravitinos with $m_{3/2} \sim 1$ TeV.

Other attractive process to produce gravitino would be the nonthermal production by the inflation decay through $\phi \rightarrow 2\psi_\mu$ [34–44] where ψ_μ means the gravitino. Since we expect a relatively low reheating temperature for the second reheating, this process can importantly affect the result because the abundance of the nonthermally-produced gravitino is inversely proportional to the reheating temperature, $Y_{3/2} \sim 2B_{3/2}Y_\phi \sim 3M_P\Gamma_{\phi \rightarrow 2\psi_\mu}/2m_\phi T_R$ where $\Gamma_{\phi \rightarrow 2\psi_\mu}$ is the differential decay width to a pair of gravitino, $B_{3/2} \equiv \Gamma_{\phi \rightarrow 2\psi_\mu}/\Gamma_\phi$ is the branching ratio into a pair of gravitinos, and Γ_ϕ is the total decay rate of the inflaton field ($\simeq T_R^2/M_P$). Adopting a result of Ref. [44], the yield of the nonthermally-produced gravitino is estimated to be

$$Y_{3/2} \sim 7 \times 10^{-15} \times \left(\frac{T_R}{10^8 \text{ GeV}} \right)^{-1} \left(\frac{\langle \phi \rangle}{10^{18} \text{ GeV}} \right)^2 \left(\frac{m_\phi}{10^8 \text{ GeV}} \right)^2, \quad (42)$$

in case of $m_\phi < \sqrt{m_{3/2} m_P}$ where $\langle \phi \rangle$ and m_ϕ are the vev and the mass of the inflaton at the decay epoch. Therefore the yield value explicitly depends on the model parameters such as $\langle \phi \rangle$ and m_ϕ , which means that we need to analyse the effect for every models. We will also discuss this type of the constraint on some models simultaneously in the following sections.

V. SOME EXAMPLES

In this section we illustrate the procedure developed in the previous section in order to compute the second reheating temperature T_{R_2} in some simple models.

A. Tree level potential

We examine a simple extension of the hilltop model studied in Ref. [20] where we consider an additional quartic term

$$V = V_0 - \frac{1}{2}|m^2|\phi^2 + \frac{\lambda}{4!}\phi^4, \quad (43)$$

that stabilises the potential and provides a minimum to support the field oscillations after inflation. The parameter λ is tuned so that the scalar potential vanishes at the minimum. With this choice, the scalar potential is written as in Eq. (6) with

$$f = \delta^2(2 - \delta)^2, \quad (44)$$

and defining $|\eta_0| \equiv |m^2|/3H_0^2$ the expectation value of the field is $\phi_v = 2|\eta_0|^{-1/2}m_P$.

In order to display the typical range of parameters allowed by observations (see Fig. 1) we fix the amount of inflation after horizon exit to the typical value $N_* \simeq 50$. Using the parametrisation $\delta_* = \delta_*(\delta_e)$ obtained from Eq. (32), the spectral index and its running are determined by δ_e and η_0 . We use Eq. (30) to plot our results in Fig. 1 in the plane η_0 - $\log_{10} C_\phi(V_0^{1/4}/m_P)$. In the $1-\sigma$ window of n_s displayed in Fig. 1 we find

$$10^{10} \text{ GeV} \lesssim H_0 \lesssim 10^{12} \text{ GeV}, \quad (45)$$

which corresponds to the range $10^7 \lesssim C_\phi \lesssim 10^8$ and $10^8 \text{ GeV} \lesssim m \lesssim 10^{10} \text{ GeV}$. Using Eq. (16) to compute the first reheating temperature T_{R_1} , we find that this ranges between

$$10^{13} \text{ GeV} \lesssim T_{R_1} \lesssim 10^{14} \text{ GeV}. \quad (46)$$

This is then far in excess of the current bounds for thermal gravitino production [29, 33]. Also, in the entire space depicted in Fig. 1 the running of the spectral index ranges between $-10^{-3} \lesssim n'_s \lesssim -10^{-4}$.

Before estimating T_{R_2} by using Eq. (24), we need to obtain an accurate estimate of the average scalar density $\langle \rho_\phi \rangle$ at late times. As mentioned before, the estimated T_{R_2} in Eq. (24) is not accurate because of two reasons. The first one owes to have assumed that the field undergoes a sudden transition to the phase of fast oscillations, whereas the second one arises because the condition that determines the onset of the fast-roll motion itself involves an order of magnitude estimate [cf. Eq. (17)]. As a result, the redshift factor (a_o/a_e) in Eq. (21) cannot be computed accurately. Using Eq. (20) and writing $H_o \simeq H_e(a_e/a_o)^2$ we obtain that Q_o is given by

$$Q_o \simeq Q_e \frac{(1 - \delta_e)^2}{(1 - \delta_o)^2} \left(\frac{a_e}{a_o} \right). \quad (47)$$

Consequently, the ratio Q_o can be determined only up to a factor of order unity, which can change the reheating temperature T_{R_2} substantially owing to its exponential dependence on Q_o [cf. Eq. (23)]. Note also that although δ_o , obtained by solving numerically Eq. (18), cannot be accurately computed either, this does not imply a significant error in Q_o because $\delta_o \ll 1$ in any case.

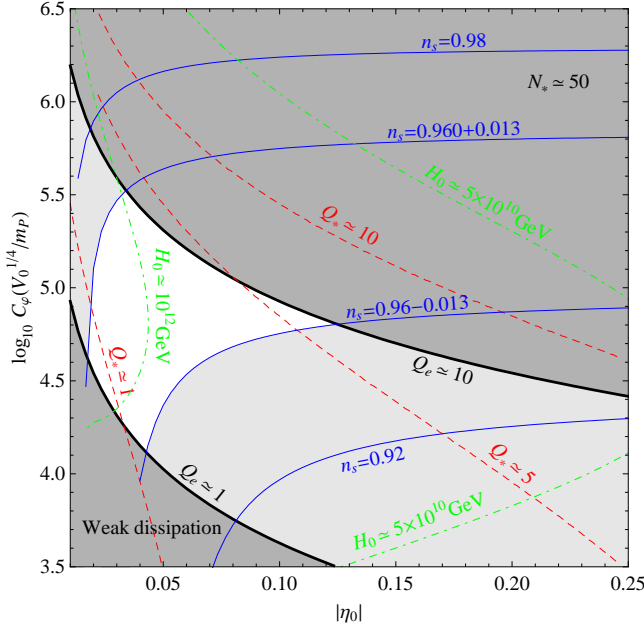


FIG. 1: Region of the parameter space allowed by observations. Heavy-shaded areas are excluded either because the inflaton field decays completely after inflation ($Q_e > 10$) or because the system is in the weak dissipation regime ($Q \lesssim 1$). The light-shaded area identifies the region where the inflaton field does not decay completely after inflation ($1 \lesssim Q_e \lesssim 10$) and the spectral index is outside its 95% CL interval. In the unshaded region the inflaton avoids its complete decay after inflation and the spectral index is within 95% CL. Dashed lines (red) correspond to constant values of the ratio Q_* , thin lines (blue) correspond to constant values of n_s and dot-dashed lines (green) correspond to constant H_0 .

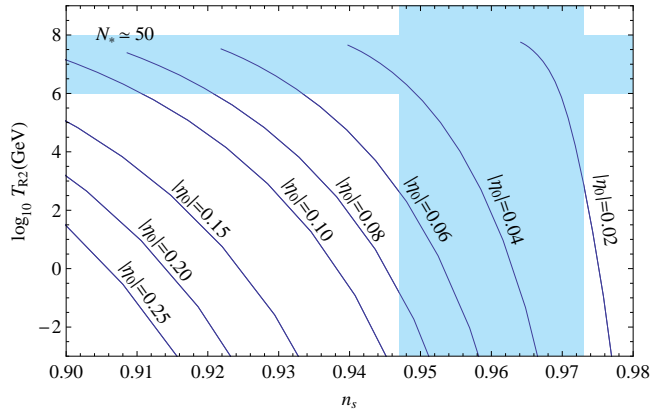


FIG. 2: Reheating temperature T_{R_2} vs spectral index n_s for several values of $|\eta_0|$. We set $m_{3/2} \sim \text{TeV}$ and $B_h = 10^{-3}$ (see Fig. 3). Only the part of the curves compatible with strong dissipation is shown. The horizontal shaded band corresponds to the range where the problem of thermal overproduction of gravitinos is less severe [29, 33]. The vertical band corresponds to the preferred range of the spectral index at $1-\sigma$.

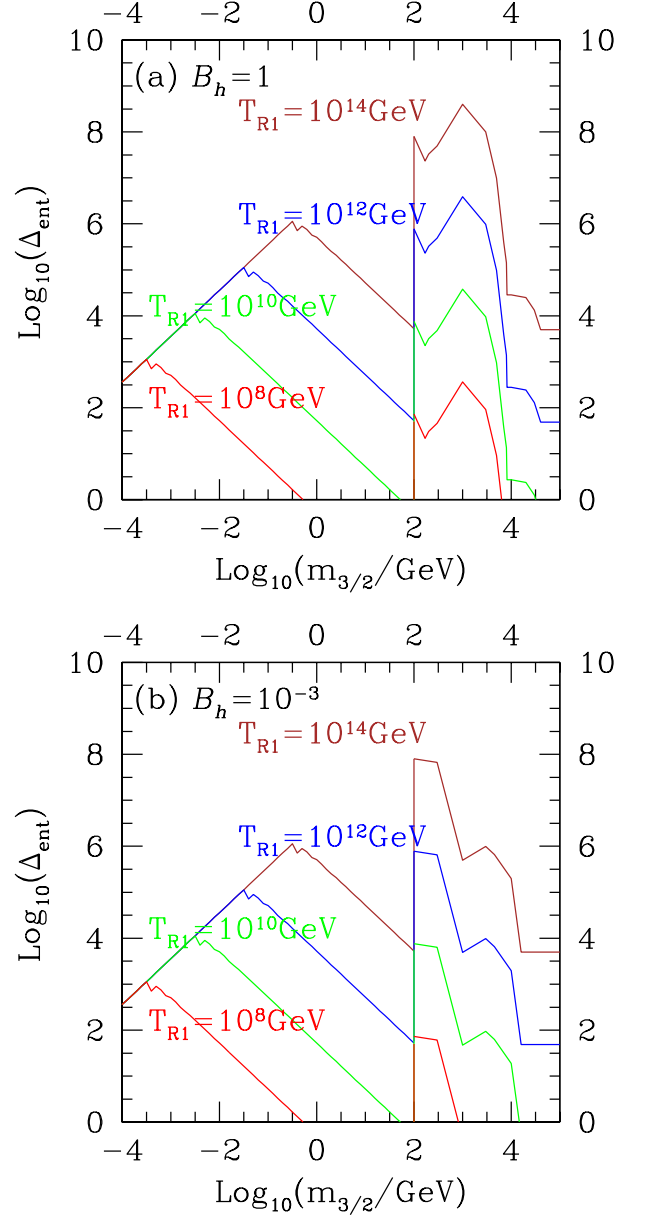


FIG. 3: Plot of $\Delta_{\text{ent}} = T_{\text{eq}}/T_{R_2}$ for a hadronic branching ratio, (a) $B_h = 1$ and (b) $B_h = 10^{-3}$, which is necessary for sufficient dilution of the gravitino abundance after the second reheating to agree with the observational constraints. From the top to the bottom, we plot the cases of $T_{R_1} = 10^{14}, 10^{12}, 10^{10}$ and 10^8 GeV, respectively.

In the Appendix we show that in order to obtain an accurate estimate for T_{R_2} it suffices to perform the transformation

$$Q_o \rightarrow \tilde{Q}_o = \alpha Q_o, \quad (48)$$

where $\alpha \simeq 1.7$. Applying such a transformation, we use

the late-time corrected densities in Eqs. (61) and (62) to estimate T_{R_2} from Eq. (24)

$$T_{R_2} \sim 2C_r^{-1/4} V_0^{1/4} \frac{e^{-3\tilde{Q}_o} f_o}{f_e^{3/4}} \left(\frac{a_o}{a_e}\right)^3 \left(\frac{a_{\text{eq}}}{a_{\text{dec}}}\right)^{3/4}. \quad (49)$$

The decay of the inflaton field must produce enough entropy as to dilute the excess of thermal gravitinos produced at temperature T_{R_1} . Given a certain reheating temperature T_{R_1} and for a certain gravitino mass $m_{3/2}$, this is achieved provided the second reheating temperature is not larger than the upper bound discussed in Sec. IV,

$$T_{R_2} = \Delta_{\text{ent}}^{-1}(m_{3/2}, T_{R_1}) T_{\text{eq}}. \quad (50)$$

Here Δ_{ent} is the entropy dilution factor and T_{eq} is the temperature of the radiation bath at the time of matter-radiation equality, which is the same order of T_{R_1} . In Fig. 2 we present our results for T_{R_2} vs n_s for several values of $|\eta_0|$ and for the typical values $N_* \simeq 50$. To obtain the curves in Fig. 2 we set $m_{3/2} \sim \mathcal{O}(\text{TeV})$ and $B_h = 10^{-3}$, hence $\Delta_{\text{ent}} \sim 10^6 (T_{R_1}/10^{14} \text{ GeV})$. Furthermore, we arrange the complete decay of the inflaton at the earliest time compatible with the dilution of the gravitino overproduction, i.e. we set $(a_{\text{eq}}/a_{\text{dec}})^{3/4} = \Delta_{\text{ent}}^{-1}$ [cf. Eqs. (24) and (50)]. If the decay of the inflaton is further delayed then T_{R_2} decreases accordingly. Note also that an excessively low reheating temperature may pose a problem when the non-thermal production of gravitinos is considered, therefore it is desirable that the inflaton field decays “soon” enough to obtain a sufficiently large reheating temperature.

The shaded areas displayed in Fig. 2 enclose the $1\text{-}\sigma$ window of n_s and the range of temperatures where the thermal gravitino overproduction are typically less problematic, i.e. $T_{R_2} \sim 10^6 \text{ GeV}$ for $B_h = 1$ and $T_{R_2} \sim 10^8 \text{ GeV}$ for $B_h = 10^{-3}$ [29, 33]. Our plots make clear that, provided $|\eta_0|$ lies in the range $0.02 \lesssim |\eta_0| \lesssim 0.05$, the inflaton, while giving rise to a thermal perturbation spectrum in agreement with current observations, manages to drive a late matter-dominated epoch and give rise to a radiation bath with temperature $T_{R_2} \lesssim 10^6 - 10^8 \text{ GeV}$ after its complete decay. This is an interesting result, and hence it is worth mentioning that the “survival” of the inflaton field owes to the moderate growth of Q during inflation. For the hilltop potential in Eq. (43), in the slow-roll regime, the dissipative ratio evolves as [7]:

$$\frac{d \ln Q}{d N_e} \simeq -\frac{|\eta_0|}{1+7Q} \left(2 - 5|\eta_0| \left(\frac{\phi}{m_P}\right)^2\right), \quad (51)$$

and thus Q starts decreasing and increases only towards the end of inflation when $\phi/m_P \sim |\eta_0|^{-1/2}$. In particular, we find $1 \lesssim Q_e/Q_* \lesssim 3.5$ within the allowed region displayed in Fig. 1. Ultimately, the fact that Υ hardly grows during inflation is due to the moderate steepness

of the potential. Therefore, to avoid the complete decay of the inflaton it is only necessary to tune the model parameters so that the last stage of inflation takes place with the system not far away from the weak dissipation limit, and therefore for not too large values of $|\eta_0|$. We note however that for the range $0.05 \lesssim |\eta_0| \lesssim 0.10$ where the gravitino overproduction is avoided, the vacuum expectation value of the field $\phi_v = 2|\eta_0|^{-1/2} m_P$ is slightly above the Planck scale.

In Fig. 3 Δ_{ent} is plotted vs $m_{3/2}$ for several values of T_{R_1} for (a) $B_h = 1$ and (b) $B_h = 10^{-3}$. For $m_{3/2} \geq 10^2 \text{ GeV}$, we adopted the constraints from the unstable gravitinos [29]. On the other hand, for $m_{3/2} < 10^2 \text{ GeV}$, we adopted the upper bound on the density of the stable gravitino not to exceed the LSP density shown in Eq. (41) by replacing m_{LSP} to $m_{3/2}$, and using Eq. (40). Then the second term in the second bracket of Eq. (40) dominates. The left-side tail with respect to the peak structure in Fig. 3 comes from the saturation of the number density due to the thermalisation of gravitino, $Y_{3/2} \lesssim 1/g_*^{3/2}$. In the current cases where $T_{R_1} \lesssim 10^{14} \text{ GeV}$ and $m_{3/2} = \mathcal{O}(\text{TeV})$, the required dilution factors are of the order of $\Delta_{\text{ent}} \gtrsim \mathcal{O}(10^8)$ and $\gtrsim \mathcal{O}(10^6)$ for $B_h = 1$ and 10^{-3} , which implies the constraint $T_{R_2} \lesssim \mathcal{O}(10^6) \text{ GeV}$ and $T_{R_2} \lesssim \mathcal{O}(10^8) \text{ GeV}$, respectively.

From (45) and $m \sim \sqrt{|\eta_0|} H_0$, we find that the inflaton masses would be in the range of $10^8 \text{ GeV} \lesssim m_\phi \lesssim 10^{10} \text{ GeV}$ because $m_\phi \sim m$ at the oscillating epoch. By considering the constraint from the nonthermal production of the gravitino given in Eq. (42) with $\langle \phi \rangle \sim m_P$ and $m_\phi \sim 10^8 \text{ GeV}$, we get $T_{R_2} \gtrsim 10^8 \text{ GeV}$ ($T_{R_2} \gtrsim 10^{10} \text{ GeV}$) for $B_h = 10^{-3}$ ($B_h = 1$). It is attractive that the gravitino abundances produced both thermally and non-thermally agree with any observational constraints if we adopt the hadronic branching ratio to be $B_h = 10^{-3}$.

The strong dissipation limit of warm inflation is known to result in the generation of Non-Gaussian effects [45–47], and several models of warm inflation have been constructed with such effects [7, 20, 48]. In such a limit it was shown in Ref. [46] that entropy fluctuations during warm inflation play an important role in generating non-Gaussianity, with the prediction

$$-15 \ln \left(1 + \frac{Q_*}{14}\right) - \frac{5}{2} \lesssim f_{\text{NL}} \lesssim \frac{33}{2} \ln \left(1 + \frac{Q_*}{14}\right) - \frac{5}{2}. \quad (52)$$

To estimate the magnitude of f_{NL} for this model within the allowed region shown in Fig. 2, it is enough to compare the results plotted there (left-hand panel) with Fig. 2. We then see that in the allowed region Q_* is at most of order 10, which corresponds to $|f_{\text{NL}}| \lesssim 10$. This is well within the observed range of the $f_{\text{NL}} = 32 \pm 21$ (68% C.L.) [27]. Note that the PLANCK satellite [49], which was launched recently, will be sensitive to non-Gaussianity at the level $|\Delta f_{\text{NL}}| = \mathcal{O}(5)$ [49, 50].

B. Supergravity inspired model

We consider now a model typical of supergravity theories [51, 52] as an example. In this kind of models, the inflaton superfield Φ is assumed to have an R charge $2/(n+1)$ allowing the superpotential

$$W_0 = -\frac{g}{n+1} \Phi^{n+1}, \quad (53)$$

with n positive and g a coupling constant. The continuous $U(1)_R$ symmetry is assumed to be dynamically broken to a discrete Z_{2nR} at a scale $v \ll m_P$ generating the superpotential

$$W_{\text{eff}} = v^2 \Phi - \frac{g}{n+1} \Phi^{n+1}. \quad (54)$$

The inflaton field $\phi(x)/\sqrt{2}$ is then identified as the real part of the scalar component of the superfield Φ . Taking the R -invariant Kähler potential, $K = |\Phi|^2 + k|\Phi|^4/4 + \dots$, the scalar potential is given by

$$V(\phi) \simeq v^4 - \frac{k}{2} v^4 \frac{\phi^2}{m_P^2} - \frac{g}{2^{n/2-1}} v^2 \frac{\phi^n}{m_P^{n-2}} + \frac{g^2}{2^n} \frac{\phi^{2n}}{m_P^{2n-4}}, \quad (55)$$

which in the absence of dissipation was shown to be flat enough to support inflation and generate the appropriate perturbation spectrum. At the vacuum, one has

$$\phi_v \simeq \sqrt{2} \left(\frac{v^2}{m_P^2 g} \right)^{1/n} m_P, \quad (56)$$

and the scalar potential is negative due to the contribution proportional to k from the non-minimal Kähler potential. In the original model [51] that was canceled by a positive SUSY breaking effect contributing an amount Λ_{SUSY}^4 , which then fixes the gravitino mass. Thus, the main role of the non-minimal Kähler k term was to fix the scale for the gravitino mass. On the other hand, by considering $k \sim g$ and $v \ll m_P$, one has in general $v \ll \phi_v$, and the quadratic term in Eq. (55) can be neglected whenever $\phi \sim \phi_v$. This is what we expect during the last 50-60 e-folds of warm inflation. As discussed earlier, warm inflation finishes giving way to a radiation dominated Universe when $\phi \lesssim \phi_v$, while the field is still in slow-roll. Owing to the slow-roll motion, it is natural to expect that $v \ll \phi \lesssim \phi_v$. Consequently, the scalar potential during the last stage of inflation can be approximated by neglecting the quadratic contribution due to the coupling k .

In the following we will just set $k = 0$ to discuss this kind of scalar potentials in the context of warm inflation. This also means that we do not consider any particular susy breaking mechanism, neither we link the gravitino mass to any particular vacuum scale. The potential Eq. (55) can be written as in Eq. (6) with $V_0 = v^4$ and

$$f(\delta) \simeq (1 - (1 - \delta)^n)^2, \quad (57)$$

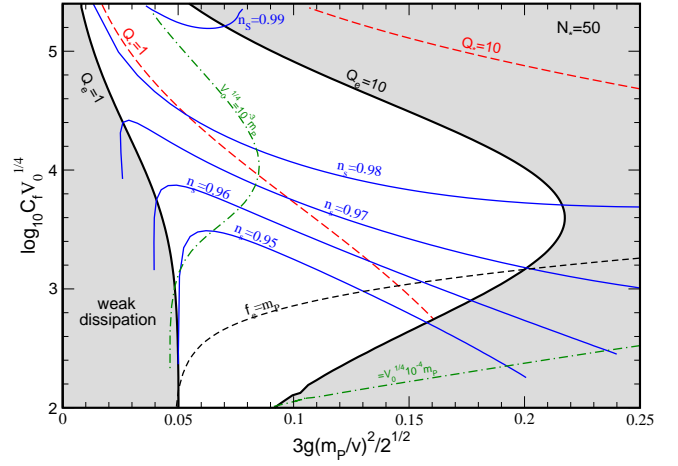


FIG. 4: Parameter space available to the strong dissipative regime of warm inflation for the sugra potential Eq. (55), with $n = 3$ and $k = 0$. In the left shaded area the system is in the weak dissipative regime ($Q_e < 1$), while the right shaded area is excluded because the inflation field decays completely after inflation ($Q_e \gtrsim 10$). The dashed lines (red) correspond to constant values of Q_* , while the thin (blue) lines are those of constant n_s . The short-dashed line gives that of constant field end value ϕ_e . Below that line the field is always subplanckian, and it is $m_P < \phi < 10m_P$ above. The dot-dashed (green) lines are those of constant vacuum energy $V_0^{1/4} = v$.

and we can apply the procedure developed in the last section to find the range of model parameters consistent with observations. For a general power n , the dissipative ratio evolves during inflation as:

$$\frac{d \ln Q}{d N_e} \simeq -\frac{|\eta_0|/(n-1)}{1+7Q} \left(14 - 6n - 5 \frac{|\eta_0|}{n-1} \left(\frac{\phi}{m_P} \right)^n \right). \quad (58)$$

When $n = 2$ we recover the potential studied in the previous subsection, with $|\eta_0| = gv^2/m_P^2$, and a moderate decrease/increase of Q during inflation. On the other hand, for steeper potentials with $n > 2$, Q always increases. Even if the observable universe exits the horizon when the system is still in the weak dissipative regime (i.e. $Q_* \lesssim 1$), inflation typically finishes into the strong dissipative regime. We are interested here on finding the model parameters which lead to a moderate value $Q_e \sim O(10)$ when $n > 2$.

Fig. 4 depicts the range of parameters allowed by observations for $n = 3$ and $N_* \simeq 50$. In this case the increase of the dissipative ratio Q is still moderate and we can find values of parameters which leads to the strong dissipative regime but with $Q_e \leq 10$, and for which the prediction for the spectral index is within the observational range. For larger powers $n \geq 4$, owing to the steepness of the potential, a large ratio Q is needed to drive slow-roll inflation. As a result, a substantial part

of the region where the spectral index agrees with observations would be excluded by the bound $f_{\text{NL}} < 74$ [27], implying $Q_* \lesssim 1.2 \times 10^4$. And in regards to the gravitino overproduction, the substantial increase of the ratio Q during the inflationary stage reduces considerably the parameter space for which $Q_e \sim O(10)$. For most of the parameter space, inflation finishes well into the strong dissipative regime with $Q_e \gg 1$. But even if we tune the parameters to avoid too large a value of Q_e , the model with $n \geq 4$ predicts a blue spectrum and no tensors. This is in conflict with observations, which in the absence of primordial tensor perturbations favour a red tilted spectrum. Nevertheless, we remark that the blue spectrum stems from the large correction $\mathcal{P}_{\mathcal{R}}^{(c)}$ to the perturbation spectrum. If one disregards such a correction, i.e. $\mathcal{P}_{\mathcal{R}}^{(c)} = 1$, the model gives rise to a red tilted spectrum with $0.956 \lesssim n_s \lesssim 0.974$, in agreement with current observations. Still, the dissipative mechanism with too step potentials does not help with the gravitino problem.

VI. CONCLUSIONS

In this paper we have shown that warm inflation models can lead to a new mechanism for controlling gravitino overproduction. The residual oscillating energy of the inflaton field after inflation eventually dominates the energy density of the universe. Then the late-time entropy production by its decay can really dilute the gravitinos which are thermally-produced in the radiation dominated epoch just after warm inflation ends. Even if we consider the nonthermal production of gravitino, this scenario is consistent with the observational constraints. We have demonstrated that this mechanism is applicable to a large class of models, when the dominant term during inflation goes like ϕ^n with $n = 2, 3$, with a mild decrease/increase of the dissipative parameter during inflation such that still $Q_e \lesssim 10$ by the end of inflation. For steeper potentials, the increase of the dissipative parameter would lead to $Q_e \gg 10$ for most of the parameter space, and the complete decay of the inflaton by the end.

We have also discussed the possibility to detect the non-Gaussianity of the order of $f_{\text{NL}} \sim 10$ which originates from the strong dissipation in the current models of warm inflation. The PLANCK satellite will be able to detect this signature by which we can distinguish the current model from the normal cold inflation models.

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VII. APPENDIX

In this section we obtain an accurate estimate for T_{R_2} . In order to do so, we need a reference time to compare the predicted scalar density after inflation, Eq. (23), with the value obtained from the numerical solution of the system, Eqs. (1) and (2). An appropriate “checkpoint” is provided by the time when the system moves into the weak dissipation regime: $\Upsilon_w = 3H_w$ (or $Q_w = 1$). This checkpoint is appropriate because when $\Upsilon = 3H$ the field is already performing fast oscillations, hence the density is well approximated by the average in Eq. (23). Also, for the cases of interest (when the inflaton does not decay completely right after inflation), the system is not far away from the weak dissipation regime. This then allows us to obtain numerically the scalar density at this time quite easily, as $\Upsilon = 3H$ not too late after the field starts oscillating.

Using Eq. (23), the predicted average scalar density when the system reaches the weak dissipation regime is

$$\langle \rho_\phi \rangle_w \simeq (\rho_\phi)_o Q_o^{-3} \exp \{ -3 Q_o (1 - Q_o^{-1}) \}, \quad (59)$$

where $a_o/a_w = Q_o^{-1}$ since Q falls as a^{-1} after inflation. Because the ratio Q_o is determined up to a factor of order 1, it is possible to match the above prediction to the numerical solution at the time $a = a_w$ by the performing the substitution

$$Q_o \longrightarrow \tilde{Q}_o = \alpha Q_o, \quad (60)$$

and then finding an appropriate value for α . Within the allowed region displayed in Fig. 1 and fixing $(\rho_\phi)_o = 2V_o$, we find that the average scalar density is matched to the numerical solution by taking $\alpha \simeq 1.70$. Hence, at late times ($a \gg a_w$) the average scalar density and the radiation density are well approximated by [cf. Eq. (23)]

$$\langle \rho_\phi \rangle \simeq (\rho_\phi)_o (\tilde{Q}_o)^{-3} \left(\frac{a_w}{a} \right)^3 \exp \{ -3 \tilde{Q}_o \}. \quad (61)$$

and

$$\rho_r \simeq 2(\rho_r)_e (\tilde{Q}_o)^{-4} \left(\frac{a_e}{a_o} \right)^4 \left(\frac{a_w}{a} \right)^4, \quad (62)$$

with the redshift factor (a_e/a_o) as given by Eq. (21). The factor of 2 in the last equation is necessary so that ρ_r matches its numerical solution at $a = a_w$. Although put by hand, the introduction of such a factor is justified because after potential-radiation equality most of the scalar density, i.e. an amount $V_e = (\rho_r)_e$, is transferred to the radiation bath in a Hubble time or so.

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